Polynomial Representations of Split-Phase Constants of Gyromagnetic Waveguides with Electric and Magnetic Walls

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***Abstract*—Two classic circular waveguides met in the design of nonreciprocal Faraday rotation devices and waveguide junction circulators are the gyromagnetic ones with electric and magnetic walls. The purpose of this letter is to present some polynomial representations of the split-phase constants in the gyrotropy of each problem region and to compare the opening between the split branches for the two situations. A polynomial representation of the cutoff space of the lower split branch of the magnetic wall problem region is separately computed.**

I. Introduction

TWO CLASSIC gyromagnetic waveguides joined in the design of nonreciprocal ferrite devices are the fully filled circular waveguide with either an electric or magnetic wall. The exact cutoff and propagation spaces of each configuration are classic solutions in the literature. Some early representative papers on the electric wall problem are given in [1]–[4] and one on the magnetic wall one is given in [5]. One distinguishable feature between the two solutions is that the magnetic wall problem region is characterized by split cutoff numbers and split-phase constants, whereas the electric wall one is associated with degenerate cutoff numbers and split-phase constants. The purpose of this letter is therefore not to revisit these solutions but to construct some polynomial representations of each in terms of the gyrotropy of the exact problem region over some practical engineering geometries. A polynomial representation of the cutoff condition of the lower split branch of the magnetic wall problem is included for completeness. While the polynomial representations summarized in this paper are valid with the gyrotropy κ bracketed between zero and unity, the practical solutions are in each case restricted by the intersections of the upper branch of the dominant split-phase constant curve and the lower one of the next higher pair. A comparison of the two solutions indicates that the splitting in the dominant pair of degenerate modes of the magnetic wall waveguide is somewhat larger than that of the electric wall one. Perturbation and other formulations valid at the origin have been separately described in [6], [10], and [14]. The two geometries considered here are depicted in Fig. 1. A knowledge of these quantities enters into the descriptions of practical Faraday rotation devices

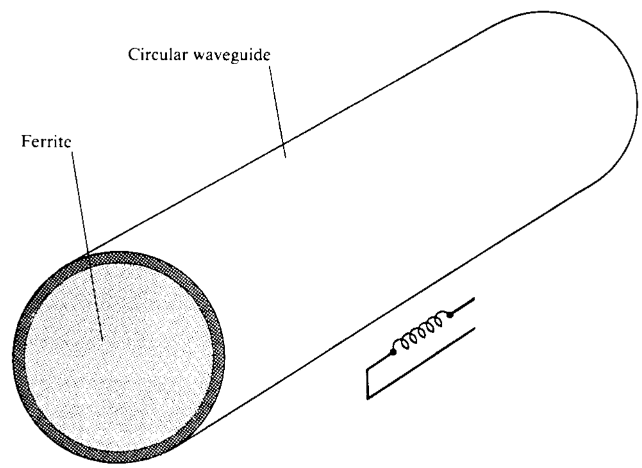

Fig. 1. Circular waveguides with electric and magnetic walls.

and conventional and re-entrant turnstile waveguide junction circulators [7]–[13]. The original waveguide circulator is a turnstile junction consisting of a junction of three rectangular waveguides with one or two circular gyromagnetic waveguides with electric walls protruding above the top and bottom walls of the structure; the re-entrant geometry consists of one or two gyromagnetic waveguides with magnetic walls in a re-entrant arrangement.

II. Calculations

A Faraday rotation section may consist either of a fully filled circular metal waveguide or an open ferrite rod in a suitable dielectric sleeve in an oversized tube. In the former case the solution reduces to a gyromagnetic problem region with an electric wall; in the latter instance one based on a magnetic wall approximation may sometimes be appropriate. The re-entrant turnstile junction circulator has its origin in the magnetic wall problem region. The geometry of a typical turnstile junction circulator employing a Faraday rotation section consisting of a magnetized ferrite rod embedded in a dielectric sleeve is depicted in Fig. 2. A knowledge of the split-phase constants of either arrangement is therefore of interest in the design of these sorts of junctions. The polynomial representations for the differential propagation constants of the dominant split modes of gyromagnetic waveguides with the direct magnetic field in the direction of propagation with

Manuscript received April 16, 1997.
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Publisher Item Identifier S 1051-8207(97)06166-7.

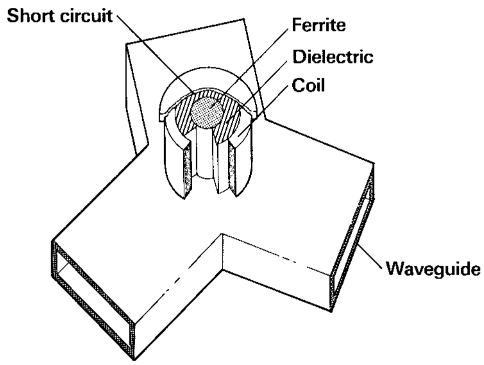

Fig. 2. Schematic diagram of waveguide turnstile junction circulator.

magnetic and electric walls given here are based on the exact descriptions in [3]–[5].

Polynomial representations for $\beta_{\pm}/k_0$ of the surface defined by k_0R and κ/μ between ± 1 based on the exact solutions of the characteristic equations of the magnetic and electric wall problem regions are

$$\frac{\beta_{\pm}}{k_0} = -11.668xy^2 + 24.657y^3 - 66.87y^2 + 61.548y + 21.809xy + 3.631yx^2 - 15.874 - 12.004x - 4.599x^2 - 1.181x^3 \quad 0.60 \le k_0R \le 0.90$$

$$\frac{\beta_{\pm}}{k_0} = -4.328xy^2 + 17.08y^3 - 45.504y^2 + 42.222y + 8.194xy - 0.647yx^2 - 10.231 - 2.635x - 0.242x^2 + 0.861x^3 \quad 0.60 \le k_0R \le 0.90$$

respectively.

The variables appearing in the polynomials are defined by

$$x = \frac{\kappa}{\mu}$$
$$y = k_0R.$$

The cutoff space of the lower split-phase constant branch of the magnetic wall problem region is as is understood dependent upon the gyrotropy. One polynomial representation for this feature which fixes the gyrotropy of the problem region in this type of waveguide is

$$\kappa/\mu = 6.694(k_0R)^3 - 18.105(k_0R)^2 + 16.841(k_0R) - 4.630, \quad \varepsilon_r = 15, \quad 0.475 < k_0R < 1.0$$
$$\kappa/\mu = 7.01(k_0R)^3 - 19.509(k_0R)^2 + 18.755(k_0R) - 5.507, \quad \varepsilon_r = 12, \quad 0.531 < k_0R < 1.0$$
$$\kappa/\mu = 6.06(k_0R)^3 - 18.022(k_0R)^2 + 18.510(k_0R) - 5.861, \quad \varepsilon_r = 10, \quad 0.582 < k_0R < 1.0.$$

This result is compared in Fig. 3 with those obtained from the corresponding perturbation and anisotropic formulations for β_+.

A scrutiny of these polynomials suggests that the opening between the split-phase constant curves is larger in a gyromagnetic waveguide with a magnetic wall than it is in one with an electric wall. The calculations entering in the description of the resonator in the re-entrant waveguide circulator described in

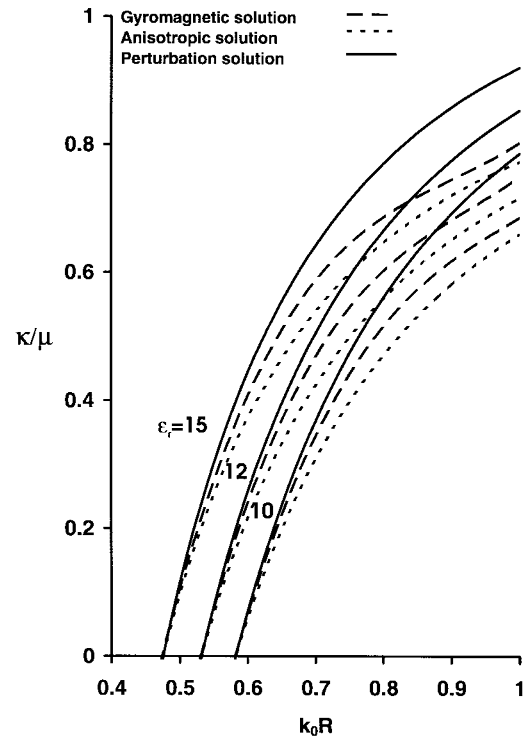

Fig. 3. Comparison between cutoff numbers of clockwise circularly polarized mode in circular waveguide with ideal magnetic wall (— — Gyromagnetic solution, - - - - Anisotropic solution, ____ Perturbation solution).

[12] should in retrospect have been based on a gyromagnetic waveguide with a magnetic boundary condition instead of an electric one. The data presented there represent therefore a lower bound on the true results.

The split phase constants of the two problem regions based on perturbation theory are separately given by

$$\beta_{\pm}^2 = k_0^2\varepsilon_f(\mu \mp C_{11}\kappa) - \left(\frac{1.84}{R}\right)^2$$

and

$$\beta_{\pm}^2 = \beta_0^2(\mu \mp C_{11}\kappa)$$

where

$$\beta_0^2 = k_0^2\varepsilon_f - \left(\frac{1.84}{R}\right)^2.$$

R is the radius of the gyromagnetic waveguide and ε_f is the relative dielectric constant of the ferrite material.

The unknown constant C_{11} in each perturbation problem region is

$$C_{11} = \frac{2}{(1.84)^2 - 1}$$

A scrutiny of these two solutions reveals a similar relationship between the openings of the split-phase constants of the two problem regions associated with the exact problem.

One way to test the robustness of the perturbation problem is to curve fit C_{11} to the exact formulation. The required result in the case of the TM_{11} gyromagnetic mode in a circular waveguide with an ideal magnetic wall with $\varepsilon_f = 15.00$ is

$$C_{11}\kappa = \Big[1 - \big[6.366y^3 - 17.266y^2 + 15.892y - 3.013xy^2 + 5.631xy + 0.938x^2y - 4.099 - 3.099x - 1.187x^2 - 0.305x^3\big]^2 - \left(\frac{0.475}{y}\right)^2\Big], \quad -1 \le \kappa \le \text{cutoff}.$$

The robustness of the perturbation formulation may now be verified by evaluating C_{11} at the origin and at $\kappa = \mp 0.50$ (say). This gives $C_{11} = 0.861$ at $\kappa = 0$, $C_{11} = 0.764$ at $\kappa = -0.50$ and $C_{11} = 0.949$ at $\kappa = 0.50$. L'Hopital's rule is used to investigate C_{11} at the origin.

The corresponding result in the case of the TE solution with $\varepsilon_f = 15.00$ is

$$C_{11}\kappa = \left(\frac{1}{1 - \left(\frac{0.475}{y}\right)^2}\right)\big[4.41y^3 - 11.749y^2 + 10.902y - 1.1175xy^2 + 2.116xy - 0.167x^2y - 2.642 - 0.68x - 0.0625x^2 + 0.2223x^3\big]^2 - 1, \quad -1 \le \kappa \le 1.$$

In the open-magnetic wall problem, below the cutoff condition, the lower split-phase constant branch may be taken as

$$\frac{\beta_+}{k_0} = 1.$$

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